Variable Structure Design of a Fault Tolerant Control System for Induction Motors

Jean-Etienne Dongmo Harry G. Kwatny[1]
Chika Nwankpa
Drexel University

Gaurav Bajpai Carole Teolis
Techno-Sciences, Inc.

Keywords: Fault Tolerant Control, Nonlinear Control, Variable Structure Control, Induction Motors.

Abstract: In this paper we describe a method for designing switching controls and analyzing achievable performance for motor drives. The method is proposed as a means for constructing a family of controls to achieve acceptable operation of motors under a variety of fault conditions.

I. INTRODUCTION

There is a growing demand for fault tolerance which can be achieved not only by improved component reliability, but also through fault detection and reconfiguration. A fault is usually understood as a component malfunction in the system that leads to undesirable overall system performance. Most fault tolerant control systems rely on redundancy of critical components may also rely on robust or adaptive control designs. The limitations of the latter approach have been noted in the literature.

Many modern motor drives, incorporate fault detection and identification [1]. However, this information is typically used to shut down the system in order to prevent further damage. Little consideration has yet been given to continued operation, albeit with reduced performance. An exception is the single phase operation of a faulted three phase machine [2]. In this work we consider induction motors in which we are primarily concerned with inverter faults.

We start with a set of fault scenarios, each scenario consisting of an individual or a combination of faults. Each scenario needs to be analyzed in terms of achievable performance and a controller that delivers maximum performance needs to be designed. The problem of control design in this setting is not trivial. Ordinarily, the control designer will have a deep understanding of the achievable performance of physical devices to be controlled. That not generally true for impaired devices. Consequently, one of our goals is to establish an analysis and design framework that can be systematically applied to each fault scenario.

In recent years there have been numerous investigations of advanced control methods for motors; particularly induction motors. The techniques applied include feedback linearization, adaptive control, and variable structure methods among others [3-5]. Feedback linearization is method of transforming a system with significant nonlinearities to an equivalent system that has a linear input output structure. Thus, linear design tools might then be applied. Not all systems are feedback linearizable, but the motors of interest to us here are. Systems that are feedback linearizable are amenable to the design of parameter adaptive and self-tuning control systems. Feedback linearization is a formalization and generalization of specialized decoupling and cancellation strategies that are used in many electro-mechanical systems. Even today, such ad hoc designs are developed without any apparent reference to the general theory, e.g. [6]. Variable structure control systems are switching control systems. They have certain useful robustness properties that make them appealing. This design approach is attractive for motor drives which are inherently switching devices. Of the many approaches to variable structure control design, the so-called sliding mode method introduced by Utkin [7] is by far the popular. This technique was first applied to motor control in the early 1980's [8, 9] and continues to be of interest, e.g. [10, 11]. The connection between feedback linearization and sliding mode control was established in [12] and the methods described therein applied to AC drives in [13]. A benefit of this approach is that it provides a systematic method to study achievable performance as well as to design controllers.

In this paper, we present a detailed analysis and synthesis of induction motor speed control is provided based on variable structure control [9, 10, 12]. Discontinuous controllers are designed for normal and impaired motors. Variable structure control design is a two step process. The first step is constructing the switching surfaces, while the second is designing the discontinuous controllers to enforce sliding. Sliding is achieved using Lyapunov design where a quadratic Lyapunov function is sought that produces a large domain of attraction.

A conventional, balanced drive is used as well as a non-conventional drive that uses three independent H-bridges to drive the stator. A comparison is made of the nominal system performance using both drives. The conventional drive is seriously affected by at least two important drive faults. The first is an open gate drive (F_2 in Figure 1) in which case the motor can be operated in single phase mode. The second is a transistor T_1 short circuit (F_3). In this case, transistor T_4 must be open and if the entire circuit can be removed (F_1) then operation with a single phase is also possible.

[1] Corresponding author:: Harry Kwatny, MEM Department, Drexel University, 32nd & Chesnut Streets, Philadelphia, PA 19104. hkwatny@coe.drexel.edu.

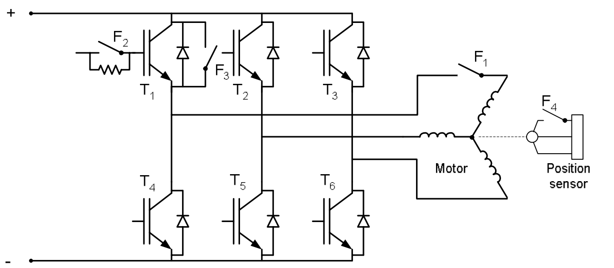

Figure 1. Conventional 3-phase drive – switches identify various faults.

The independent H-bridge configuration allows two phase operation in similar circumstances. Variable structure controllers are designed and used to examine achievable performance for one and two phase operations.

II. Control Design

The details of our approach to control design can be found in [12, 14]. We briefly summarize the essential ideas. We assume that the system is a nonlinear dynamical system described by

$$\dot{x} = f(x) + G(x)u \tag{1}$$

where $x \in R^n$ is the state and $u \in R^m$ is the control. In addition, we assume that there are precisely m output variables $y \in R^m$ that we wish to regulate,

$$y = h(x) \tag{2}$$

In particular we seek a state feedback control such that the closed loop system is stable and $y(t) \to 0$ as $t \to \infty$. Some systems defined by (1) and (2), including those we are interested, in are input-output linearizable by a state transformation and feedback. In this case there exists a transformation $x \mapsto (\xi, z)$ such that the system equations transform to

$$\begin{aligned} \dot{\xi} &= F(\xi, z, u) \\ \dot{z} &= Az + B\left[\alpha(x) + \rho(x)u\right] \\ y &= Cz \end{aligned} \tag{3}$$

where $\rho(x)$ is an $m \times m$ nonsingular matrix. In fact, the transformation can be constructed so A, B, C are in Brunovsky form. The first equation is referred to as the 'internal dynamics.'

Clearly, one can introduce a new control variable v and choose $u = \rho^{-1}(-\alpha + v)$ so that the relation between the input v and output y is linear. We will not do this here. Instead, we will choose a switching control,

$$u_i = \begin{cases} u_i^+(x) & s_i(x) > 0 \\ u_i^-(x) & s_i(x) < 0 \end{cases} \quad i = 1, \ldots, m \tag{4}$$

So that each control u_i switches across its switching surface $s_i = 0$ between the smooth functions $u_i^+(x), u_i^-(x)$. It is possible that closed loop trajectories exist in the intersection of the switching surfaces, ie., within the set $s(x) = 0$ These are called sliding modes. This set is called the 'sliding manifold.' One design approach forces sliding modes to occur [7]. Behavior in the sliding mode depends on the structure of the switching surface.

The method proposed in [12] defines the switching surfaces by

$$s(x) = Kz(x) \tag{5}$$

and proposes one way to define K that insures all sliding trajectories converge to a subset of the switching manifold that corresponds to $y \equiv 0$.

Once the switching surfaces are designed it is necessary to specify $u_i^+(x), u_i^-(x)$ to force all trajectories to enter the sliding manifold. This is accomplished with a Lyapunov analysis. Define the function

$$V(s) = s^T Q s \tag{6}$$

And compute using (3) and (5)

$$\dot{V} = 2\left[KAz + \alpha\right]^T QKz + 2u^T \rho^T QKz \tag{7}$$

If we further assume that the control is bounded, say $|u_i| \le \bar{U}_i$, then we can minimize $\dot{V}$ by choosing

$$u_i = -\bar{U}_i \operatorname{sign}\left(s_i^*(x)\right), \quad s^*(x) = \rho^T(x) QKz(x) \tag{8}$$

Then $\dot{V} < 0$ provided

$$\left|\bar{U}^T \rho^T QKz\right| > \left|\left[KA + \alpha\right]^T QKz\right| \tag{9}$$

III. Induction Motor Analysis

Consider a 3-phase induction motor with two field windings. With the conventional drive topology, the balanced operation condition $i_1 + i_2 + i_3 = 0$ is enforced, in which case the motor dynamics in Blondel-Park coordinates are

$$\begin{bmatrix} J & 0 & 0 & 0 & 0 \\ 0 & L_s & 0 & L_{fd} & 0 \\ 0 & 0 & L_s & 0 & L_{fd} \\ 0 & L_{fd} & 0 & L_f & 0 \\ 0 & 0 & L_{fd} & 0 & L_f \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \omega \\ i_d \\ i_q \\ i_{fd} \\ i_{fq} \end{bmatrix} + \begin{bmatrix} 0 & -L_{fd} i_{fd} & L_{fd} i_{fd} & 0 & 0 \\ L_{fd} i_{fd} & r & \omega L_s & 0 & 0 \\ -L_{fd} i_f & -\omega L_s & r & 0 & 0 \\ 0 & 0 & 0 & r_f & 0 \\ 0 & 0 & 0 & 0 & r_f \end{bmatrix} \begin{bmatrix} \omega \\ i_d \\ i_q \\ i_{fd} \\ i_{fq} \end{bmatrix} = \begin{bmatrix} -\tau \\ v_d \\ v_q \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

The rotor and stator flux are given by

$$\psi_{r,d} = L_f i_{fd} + L_{fd} i_d, \quad \psi_{r,q} = L_f i_{fq} + L_{fd} i_q \tag{11}$$

$$\psi_{s,d} = L_s i_d + L_{fd} i_{fd}, \quad \psi_{s,q} = L_s i_q + L_{fd} i_{fq} \tag{12}$$

The electrical torque is

$$T = L_{fd} i_{fd} i_d - L_{fd} i_{fq} i_q \tag{13}$$

ω denotes the rotor speed and τ the mechanical load torque. Let us consider the constant speed operation of the motor, $\omega = \omega_0$ and $\tau = \tau_0$. The 3 stator windings are supplied with alternating current at frequency ω_s and separated in phase by $2\pi/3$ radians. A stator electromagnetic field is established of constant magnitude and it rotates about the motor axis at a frequency of ω_s .The rotor rotates at a speed $\omega < \omega_s$ so that an alternating current is induced in the rotor coils at a frequency of $\Delta\omega = \omega_s - \omega$. In this way a rotor electromagnetic field is established that rotates at a speed of $\Delta\omega$ relative to the rotor or at a speed of ω_s relative to inertial space. Consequently, the stator and rotor electromagnetic fields rotate at the same speed in inertial space, separated by a constant phase angle. The d,q- axis and rotor currents and voltages vary periodically in time with a frequency of $\Delta\omega$.

In designing a speed regulator we want to achieve the steady state operation described above. We wish to regulate the rotor speed $\omega \to \omega_0$. However, because we have multiple control inputs, v_d, v_q, v_0, or equivalently, v_1, v_2, v_3, we can regulate additional variables as well. Consequently it makes sense to regulate $i_0 \to 0$ to force balanced operation and also the electromagnetic flux magnitude $\|\psi\| \to \psi_0$. In the case of the conventional drive, balance operation is enforced by the topology of the drive circuitry, and there are only two independent voltages. So, we have 2 regulated outputs:

$$y_1 = h_1(\omega, i_d, i_q, i_f) := \omega - \omega_0$$

$$y_2 = h_2(\omega, i_d, i_q, i_f) := \|\psi\| - \psi_0 = \sqrt{\left(L_f i_{fd} + L_{fd} i_d\right)^2 + \left(L_f i_{fq} + L_{fd} i_q\right)^2} - \psi_0$$

To facilitate an understanding of motor operation, we note that the partial transformation $x \mapsto z$ is defined by

$$z(x) = \begin{bmatrix} \omega - \omega_0 \\ \dfrac{L_{fd} i_{fd} i_d - L_{fd} i_{fq} i_q}{J} \\ \sqrt{\left(L_f i_{fd} + L_{fd} i_d\right)^2 + \left(L_f i_{fq} + L_{fd} i_q\right)^2} - \Psi_0 \\ -\dfrac{\left(i_{fd}\left(L_f i_{fd} + L_{fd} i_d\right) + i_{fq}\left(L_f i_{fq} + L_{fd} i_q\right)\right) r_f}{\sqrt{\left(L_f i_{fd} + L_{fd} i_d\right)^2 + \left(L_f i_{fq} + L_{fd} i_q\right)^2}} \end{bmatrix} \quad (14)$$

If $y(t) \equiv 0$, as we require, it must be that $z(x) \equiv 0$. Using (14) this condition leads to the following result [12]: along trajectories satisfying $y(t) \equiv 0$, the rotor flux satisfies the differential equation

$$\frac{d}{dt}\begin{bmatrix} \psi_{rd} \\ \psi_{rq} \end{bmatrix} = \frac{r_f \tau}{\psi_0}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \psi_{rd} \\ \psi_{rq} \end{bmatrix} \quad (15)$$

With initial conditions satisfying

$$\psi_{rd}^2(0) + \psi_{rq}^2(0) = \psi_0^2 \quad (16)$$

We also obtain the rotor and stator currents as a function of the flux components:

$$\begin{bmatrix} i_{fd} \\ i_{fq} \end{bmatrix} = \frac{1}{\psi_0}\begin{bmatrix} 0 & \tau \\ -\tau & 0 \end{bmatrix}\begin{bmatrix} \psi_{fd} \\ \psi_{fq} \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \frac{L_f}{L_{fd}\psi_0}\begin{bmatrix} \psi_0/L_f & \tau \\ -\tau & \psi_0/L_f \end{bmatrix}\begin{bmatrix} \psi_{fd} \\ \psi_{fq} \end{bmatrix} \quad (18)$$

Thus, the flux and current components are all sinusoidal in time. From (15) and (16) we see that the flux vector rotates with a speed $\Delta\omega = r_f \tau / \psi_0$ relative to the rotor, and from (17) and (18) we further conclude that the rotor current phasor leads the stator current phasor by an angle of $\tan\left(r_f \tau / \psi_0\right)$.

These results confirm our understanding of how the unimpaired motor operates, but the same technique can be used for the faulted motor.

IV. Motor Control

The system dynamics (10) can be written in the form (1) with

$$f(x) = \begin{bmatrix} \dfrac{L_{fd} i_{fd} i_q - L_{fd} i_{fq} i_d}{J} \\ \dfrac{L_f\left(L_s i_q + L_{fd} i_{fq}\right) - L_f r i_d + L_{fd} r_f i_{fd}}{L_f L_s - L_{fd}^2} \\ \dfrac{-L_f\left(L_s i_d + L_{fd} i_{fd}\right) - L_f r i_q + L_{fd} r_f i_{fq}}{L_f L_s - L_{fd}^2} \\ \dfrac{-L_{fd}\left(L_s i_q + L_{fd} i_{fq}\right) + L_{fd} r i_d - L_s r_f i_{fd}}{L_f L_s - L_{fd}^2} \\ \dfrac{L_{fd}\left(L_s i_d + L_{fd} i_{fd}\right) + L_{fd} r i_q - L_s r_f i_{fq}}{L_f L_s - L_{fd}^2} \end{bmatrix} \quad (19)$$

$$G(x) = \begin{bmatrix} 0 & 0 \\ \dfrac{-L_f L_{fd}^2 + L_f^2 L_s}{-2L_s L_f L_{fd}^2 + L_{fd}^4 + L_s^2 L_f^2} & 0 \\ 0 & \dfrac{L_s L_f^2 - L_{fd}^2 L_f}{-2L_s L_f L_{fd}^2 + L_{fd}^4 + L_s^2 L_f^2} \\ \dfrac{L_{fd}^3 - L_f L_{fd} L_s}{-2L_s L_f L_{fd}^2 + L_{fd}^4 + L_s^2 L_f^2} & 0 \\ 0 & \dfrac{-L_s L_f L_{fd} + L_{fd}^3}{-2L_s L_f L_{fd}^2 + L_{fd}^4 + L_s^2 L_f^2} \end{bmatrix} \quad (20)$$

Where the state and control are

$$x = \begin{bmatrix} \omega & i_d & i_q & i_{fd} & i_{fq} \end{bmatrix}^T$$
$$u = \begin{bmatrix} v_d & v_q \end{bmatrix}^T \quad (21)$$

We wish to implement the switching control with the stator voltages v_1, v_2, v_3 rather than the Blondel-Park voltages, so we use the we use the Blondel-Park transformation and the fact that $v_0 = 0$ to derive the replacement relation

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = B^* \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad B^* = \begin{bmatrix} \sqrt{\tfrac{2}{3}}\cos\theta & -\dfrac{\cos\theta}{\sqrt{6}} + \dfrac{\sin\theta}{\sqrt{2}} & -\dfrac{\cos\theta}{\sqrt{6}} - \dfrac{\sin\theta}{\sqrt{2}} \\ \sqrt{\tfrac{2}{3}}\sin\theta & -\dfrac{\cos\theta}{\sqrt{2}} - \dfrac{\sin\theta}{\sqrt{6}} & \dfrac{\cos\theta}{\sqrt{2}} - \dfrac{\sin\theta}{\sqrt{6}} \end{bmatrix}$$

Moreover, because we now have a θ-dependence we add the equation $\dot{\theta} = \omega$ to the system defined by (19) and (20). The controller (8) is computed as

$$v_i = V \operatorname{sign} s_i^*,\ s^*(\theta, \omega, i_d, i_q, i_{fd}, i_{fq}) = \left(B^*\right)^T \rho^T Q K z \quad (22)$$

Where z and B^* are given above, and

$$\rho = \begin{bmatrix} \dfrac{L_{fd}\left(L_f i_{fq} + L_{fd} i_q\right)}{J\left(L_{fd}^2 - L_s L_f\right)} & -\dfrac{L_{fd}\left(L_f i_{fd} + L_{fd} i_d\right)}{J\left(L_{fd}^2 - L_s L_f\right)} \\ \dfrac{-L_{fd}\left(L_f i_{fd} + L_{fd} i_d\right) r_f}{\left(L_{fd}^2 - L_s L_f\right)\sqrt{\left(L_f i_{fd} + L_{fd} i_d\right)^2 + \left(L_f i_{fq} + L_{fd} i_q\right)^2}} & \dfrac{-L_{fd}\left(L_f i_{fq} + L_{fd} i_q\right) r_f}{\left(L_{fd}^2 - L_s L_f\right)\sqrt{\left(L_f i_{fd} + L_{fd} i_d\right)^2 + \left(L_f i_{fq} + L_{fd} i_q\right)^2}} \end{bmatrix}$$

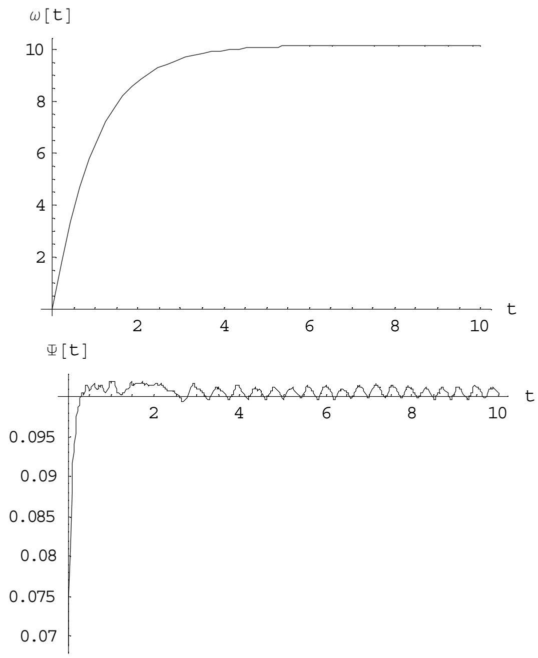

Figure 3. The primary controlled variables, speed and rotor flux magnitude.

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}, K = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 10 & 1 \end{bmatrix}, V = 30$$

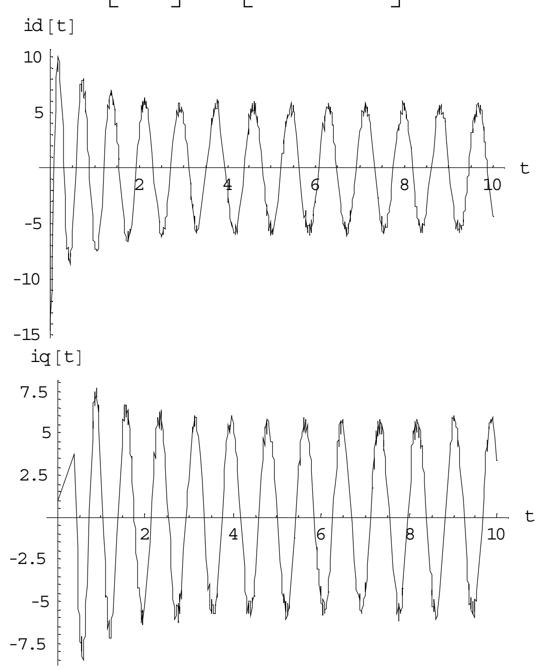

Figure 2. The stator currents.

V. SIMULATIONS

The system with the controller described above has been simulated and we show some of the results. The data is as follows:

$$J = 0.0433, L_s = 0.065, L_f = 0.06$$
$$r_f = 0.15, \tau = 0.5, \omega_0 =$$

Figure 3 shows the two regulated variables, speed and rotor flux magnitude. Note that the sliding behavior for the speed has been specified to have a 1 sec time constant whereas the sliding time constant for rotor flux has been specified to be 0.1 sec.

Figure 2 shows the stator currents in the Blondel-Park coordinates i_d, i_q. We see the anticipated sinusoidal waveforms. The rotor currents show the same behavior.

Figure 4 and Figure 5 show the sliding surfaces.

$$s(x) = Kz(x)$$

It is seen that sliding is achieved quite rapidly.

VI. CONCLUSIONS

This paper describes a general procedure for designing switching controllers for motors. The methods proposed can be used to investigate the achievable performance and to design controllers for intact and impaired motors Here we have illustrated the process for the speed control of unimpaired inductions motors. It applies equally well to permanent magnet or synchronous motors.

Other studies have been conducted for induction motors with a single rotor winding, with independent control of all three phase windings, and for various parameters and initial conditions. Current work addresses a variety of impaired motor cases.

The investigations to date implement controllers with the assumption of the availability of all states. Clearly, the rotor currents are not available so an estimator needs to be employed.

VII. REFERENCES

[1] S. Nandi, H. A. Tolyat, and X. Li, "Condition Monitoring and Fault Diagnosis of Electrical Motors - A Review," *IEEE Transactions on Energy Conversion,* vol. 20, pp. 719-729, 2005.

[2] D. Kastha and B. K. Bose, "Fault Mode Single-Phase Operation of a Variable Frequencey Induction Motor Drive and Improvement of Pulsating Torque Characteristics " *IEEE Transactions on Industrial Electronics,* vol. 41, pp. 426-433, 1994.

[3] A. Isidori, *Nonlinear Control Systems*, 3 ed. London: Springer-Verlag, 1995.

[4] H. G. Kwatny and G. L. Blankenship, *Nonlinear Control and Analytical Mechanics*. Boston: Birkhauser, 2000.

[5] H. Nijmeijer and H. J. van der Schaft, *Nonlinear Dynamical Control Systems*. New York: Springer–Verlag, 1990.

[6] M. Boussak and K. Jarray, "A High-Performance Sensorless Indirect Stator Flux Orientation Control of

Induction Motor Drive," *IEEE Transactions on Industrial Electronics,* vol. 53, pp. 41-49, 2006.

[7] V. I. Utkin, *Sliding Modes and Their Application*. Moscow: MIR, 1974 (in Russian) 1978 (in English).

[8] A. Sabanovic and D. Izosomov, "Application of Sliding Modes to Induction Motor Control," *IEEE Transactions on Industry Applications,* vol. IA-17, pp. 41-49, 1981.

[9] D. Izosomov and V. I. Utkin, "Sliding Mode Control of Electric Motors," in *VIII IFAC Congress*, Kyoto, 1981, pp. 13-19.

[10] V. Utkin, J. Guldner, and J. Shi, *Sliding Mode Control in Electromechanical Systems*. Philadelphia: Taylor and Francis, 1999.

[11] W.-J. Wang and J.-Y. Chen, "Passivity-Based Sliding Mode Position Control for Induction Motors," *IEEE Transactions on Energy Conversion,* vol. 20, pp. 316-321, 2005.

[12] H. G. Kwatny and H. Kim, "Variable Structure Regulation of Partially Linearizable Dynamics," *Systems & Control Letters,* vol. 15, pp. 67–80, 1990.

[13] H. G. Kwatny, "Variable Structure Control of AC Drives," in *Variable Structure Control for Robotics and Aerospace Applications*. vol. 10, K. D. Young, Ed. Amsterdam: Elsevier, 1993.

[14] H. G. Kwatny and G. L. Blankenship, *Nonlinear Control and Analytical Mechanics: a computational approach*. Boston: Birkhauser, 2000.

This work was supported in part by the Office of Naval Research under contract # N00014-06-M-0262

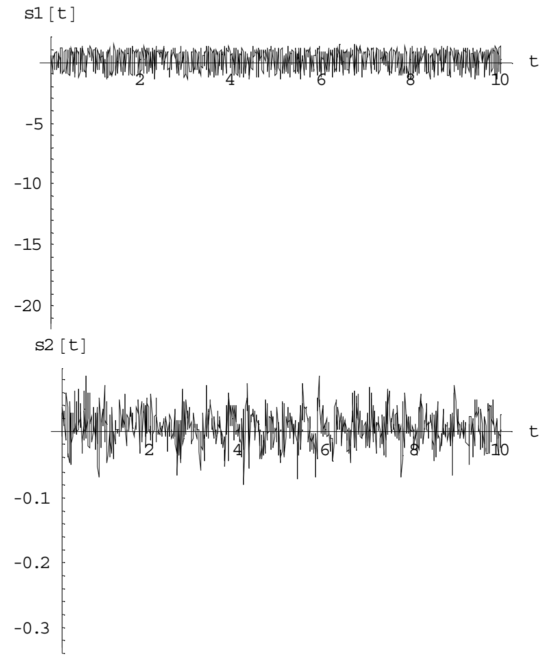

Figure 4. The sliding surfaces - long time scale.

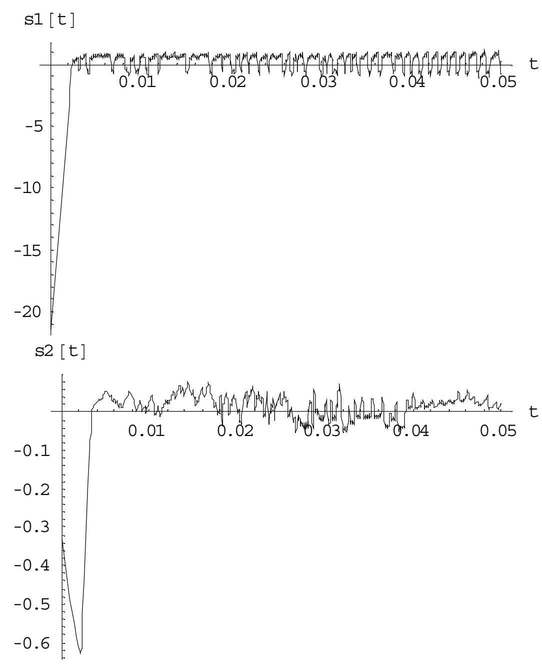

Figure 5. The sliding surfaces - short time scale.